AN ANALYSIS OF THE J-DISCONTINUITY IN SCATTERED X-RAYS—PART-III

HIRENDRA KUMAR PAL

Department of Pure Physics, Calcutta University, Calcutta.

(*Received May 31, 1965*)

ABSTRACT. The paper discusses the cause of a discontinuity of the second kind which constitutes a significant feature of the J-phenomenon. It has been shown that the occurrence of this discontinuity and the associated characteristics may be accounted for in terms of an irregular fluctuation of the percentage modification during scattering. A possible interpretation has also been suggested for the 'levels' of X-ray activity postulated by Barkla.

INTRODUCTION

In two previous papers, Parts I and II (Pal, 1964 and 1965) an attempt had been made to give an interpretation of the J-discontinuity of one kind and some aspects of the J-phenomenon, on the basis of established theory and simple acceptable assumptions In the present paper, the other aspects, with particular reference to the J-discontinuity of a second kind are studied analytically The experiments on the J-phenomenon with which the present analysis is concerned were performed in the following way : Keeping the penetrating power of the incident X-radiation constant the scattered beam in a particular direction ϕ was compared either to the primary beam or to another scattered beam in a different direction θ. For this purpose, the two beams compared were each passed through equal thickness x of any absorbing substance (generally Al) and the corresponding ratio of ionisations $(S'/P')_{90°}$ or S'_θ/S'_φ due to the intercepted beams determined. as they were progressively filtered, and plotted against increasing thickness x. The experiments were conducted by Barkla in collaboration with Khastgir (1925 a,b) Watson (1926), Mackenzie (1926 a, b), Sen Gupta (1929), Kay (1933) and others including the writer (1935-37). Khastgir, Watson and Kay compared the scattered radiation at an angle 90° with the primary, whereas Mackenzie compared the two scattered beams at angles 60° and 120° respectively. When the graph $(S'/P')_{90°}$ or $S'_{120°}/S'_{60°}$ plotted against x, was drawn, it was found, in general, to be *either*, a continuous curve with a descending course *or*, a graph (which might be horizontal i.e. parallel to the x-axis) showing one or more discontinuities (i.e. steps).

The discontinuities observed in the latter alternative were also termed J-discontinuities and they occurred seemingly at definite critical value of the mass-

*Present Address, A-91, H. B. Town, P. O. Sodpur, 24 Parganas, W. B. India.

absorption coefficient $(\mu/\rho)_{Al}$, of the transmitted radiation. Each discontinuity (J_1, J_2, J_3 ...), as postulated by Barkla, signified a transformation from one "level' of X-ray activity to another, there being altogether several such (absorption) levels and such a transformation was called by him the J-transformation. It was not, however, possible to predict which of the two alternative types of graph mentioned above, one was going to get; for, strangely, the discontinuity occurred and did not occur under what appeared to be identical experimental conditions. For a comprehensive perusal, the reader is especially referred to the original papers by Barkla *et al.*

THEORETICAL

A. *The ratio* $(S'/P')_{90°}$:

In the previous paper, Part II, (Pal, 1965) it was shown that

$$(S'/P')_{90^\circ} = K'\left[1-3C'_{90^\circ}\left\{\frac{B}{2}\, x\lambda^3\left(r_x+\frac{\delta\lambda_{90^\circ}}{\lambda}\right)+\left(\frac{\delta\lambda_{90^\circ}}{\lambda}-r_x\right)\right\}+(A-A')x\right] \quad \ldots \quad (1)$$

where

K' = a constant depending on the scattering material.

$C'_{90°}$ = percentage modification of the scattered rays in the direction $\phi = 90°$.

A, B = constants depending on the absorbing element.

A' = the modified value of A (for the scattered rays) depending on the geometry of the apparatus.

λ = average incident wavelength.

$\delta\lambda_{90°}$ = the Compton-change of wavelength for $\phi = 90°$.

x = thickness of the absorber.

and r_x = 'disparity' -term for $\phi = 90°$.

Further, when the ratio $(S'/P')_{90°}$ is plotted against small values of x, for Al -absorber, the graph should (if $C_{90°}$ is not zero) be a descending curve. Experimentally also, when the J-discontinuity was not in evidence, this was the type met with. In exceptional circumstances, however, as in the case of Ag- or Sn-absorber, the graph may sometimes be horizontal straight lines parallel to the x-axis.

B. *The ratio* $S'_{120°}/S'_{60°}$:

In the previous paper referred to above (Pal, 1965), it was also shown that by neglecting absorption inside the scatterer, we can write

$$(S'/P')_\varphi = K'f(\phi, \lambda)[\exp\{(A-A')x\}]\left[1+C_\varphi \frac{f'_\varphi}{f_\varphi}\,\delta\lambda_\varphi\right.$$

$$\left.-\frac{3B}{2}\, x\lambda^3\left\{\frac{\Delta_\varphi}{\lambda}+p_x+C_\varphi\left(q_x-p_x+\frac{\delta\lambda_\varphi}{\lambda}\right)\right\}+3\left\{p_x+C_\varphi\left(q_x-p_x-\frac{\delta\lambda_\varphi}{\lambda}\right)\right\}\right] \ldots (2)$$

where

$$f_\varphi = f(\phi, \lambda) = (1+\cos^2\phi)\{1+\psi(\phi, \lambda)\}$$

and $\{1+\psi(\phi, \lambda)\}$ = a factor representing the enhancement of the scattered intensity due to interference.

Δ_φ = increase in the scattered wavelength in the direction ϕ, due to interference

$p_x \cdot q_x$ = the 'disparity'—terms corresponding respectively to interference and to the combination of the Compton effect and the interference effect at an angle ϕ. The other terms are as previously explained.

If there is no interference at the scattering angle θ then $\psi(\theta, \lambda) = 0$, and $f(\theta, \lambda) = 1+\cos^2\theta$

Also $$f'_\theta = \frac{\partial f_\theta}{\partial \lambda} = 0, \qquad \Delta_\theta = 0 \text{ and } p_x = 0.$$

so that from (2) we have

$$(S'/P')_\theta = K'f(\theta, \lambda)[\exp\{(A-A')x\}]\left[1-\frac{3B}{2}x\lambda^3\left\{C'_\theta\left(s_x+\frac{\delta\lambda_\theta}{\lambda}\right)\right\}\right.$$

$$\left.-3\left\{C'_\theta\left(s_x-\frac{\delta\lambda_\theta}{\lambda}\right)\right\}\right] \quad \ldots \quad (3)$$

where s_x = the 'disparity'-term due to the Compton effect in the direction θ.

Now putting $\phi = 60°$ and $\theta = 120°$.

we get, $f(\phi, \lambda) = (1+\cos^2 60°)\{1+\psi(60°, \lambda)\} = 5\{1+\psi(60°, \lambda)\}/4$

and $f(\theta, \lambda) = (1+\cos^2 120°) = 5/4$

so that dividing (3) by (2) and substituting the values of $f(\phi, \lambda)$ and $f(\theta, \lambda)$ we get approximately

$$S'120°/S'60° = K''\left[1-C'_\varphi\frac{f'_\varphi}{f_\varphi}\delta\lambda_\varphi - \frac{3B}{2}x\lambda^3 M(x)+3\{M(x)-N\}\right] \quad \ldots \quad (4)$$

where $K'' = \{1+\psi(60°, \lambda)\}^{-1}$ = Const. (independent of x).

$$M(x) = \left\{C_\theta\left(s_x+\frac{\delta\lambda_\theta}{\lambda}\right) - \frac{\Delta_\theta}{\lambda} - p_x - C_\varphi\left(q_x - p_x + \frac{\delta\lambda_\varphi}{\lambda}\right)\right\}$$

and $N = 2C'_\theta\frac{\delta\lambda_\theta}{\lambda} - \frac{\Delta_\varphi}{\lambda} - 2C'_\varphi\frac{\delta\lambda_\varphi}{\lambda}$ = const. (independent of x).

When the ratio $S'120°/S'60°$ is plotted against x, the course of the graph will be determined by $x.M(x)$ and $M(x)$. In the case of a light scatterer like paraffin wax, p_x is small and $M(x)$ is positive ($\delta\lambda_\theta > \delta\lambda_\varphi$ and $s_x > q_x$). Also, $x \cdot M(x)$

increases initially and $M(x)$ diminishes, as x increases. This means that the ratio falls initially with an increase in x. Hence in such a case a curve descending in the direction of increasing x is expected. This is corroborated by graph No. 13, p. 545 of the paper by Barkla and Mackenzie (1926a).

The unintercepted ratio may be deduced from equation (4) by putting $x = 0$. Thus

$$S_{120^\circ}/S_{60^\circ} = K''\left[1 - C'_\varphi \frac{f'_\varphi}{f_\varphi}\,\delta\lambda_\varphi + 3\left\{M(0) - N\right\}\right] \qquad \dots \quad (5)$$

where

$$M(0) = 2\left(C'_\theta \frac{\delta\lambda_\theta}{\lambda} - \frac{\Delta_\varphi}{\lambda} - C'_\varphi \frac{\delta\lambda_\varphi}{\lambda} \right) = \text{const.} \qquad \dots \quad (6)$$

In Barkla and Mackenzie's experiments, the course of the graphs depicting the discontinuities was, however generally straight and horizontal.

The conditions for the horizontal graph may be deduced as follows : For horizontality the ratio S'_{120}/S'_{60} should be independent of x. This requires that $M(x)$ in equation (4) should be equal to zero

$$\text{i.e.,}\quad -\{C'_\theta s_x - p_x - C'_\varphi(q_x - p_x)\} = C'_\theta \frac{\delta\lambda_\theta}{\lambda} - \frac{\Delta_\varphi}{\lambda} - C'_\varphi \frac{\delta\lambda_\varphi}{\lambda} = \alpha \text{ (const.)}$$

for all values of x concerned. Clearly this identity is fulfilled if the constant α is equal to 0; i.e.,

$$C'_\theta \delta\lambda_\theta - \Delta_\varphi - C'_\varphi \delta\lambda_\varphi = 0 \qquad \dots \quad (7)$$

and

$$C'_\theta s_x - p_x - C'_\varphi(q_x - p_x) = 0 \qquad \dots \quad (8)$$

for all values of x concerned.

The relations (7) and (8) provide the conditions necessary for a horizontal graph. It is, however, noted that (8) transforms into (7) when $x = 0$. Hence the general condition for the horizontality is contained in (8) which must be true for all values of x concerned, from zero upward.

From equations (6) and (7) the value of $M(0)$ is found to be equal to 0 — (when the graph is horizontal).

Corollary 1. *For large values of p_x the condition* (8) *reduces to*

$$p_x \approx C_\theta \cdot s_x$$

Corollary 2. *When p_x is negligible, i.e. when interference is absent, the condition* (8) *cannot be satisfied and there can be no horizontal graph then.*

Corollary 3. *When $x \gtreqless x_c$ (corresponding to s_x), the condition* (8) *is automatically satisfied; for, each term then is separately equal to zero; and* (7) *is the surviving condition which should be fulfilled.*

Referring to equation (5), the constant ratio for the horizontal graph is obtained by substituting the value of $M(0)$ and N in that equation. Thus

$$S'120^\circ/S'60^\circ = S120^\circ/S60^\circ = K''\left[1 - C_\varphi \frac{f'_\varphi}{f_\varphi}\,\delta\lambda_\varphi - 3\,\frac{\Delta_\varphi}{\lambda}\right] \quad \ldots \quad (9)$$

Now $\quad \delta\lambda_\varphi = \delta\lambda 90^\circ.\ \text{vers}\ 60^\circ = \delta\lambda 90^\circ/2$

and $\quad \delta\lambda_0 = \delta\lambda 90^\circ.\ \text{vers}\ 120^\circ = 3\delta\lambda 90^\circ/2 \quad \ldots \quad (10)$

Substituting these values of $\delta\lambda_\varphi$ and $\delta\lambda_0$ in eqn. (7)

$$\Delta_\varphi = (3C_0 - C_\varphi)\delta\lambda 90^\circ/2 \quad \ldots \quad (11)$$

From (9), (10) and (11) the constant ratio can also be written down as :

$$S'120^\circ/S'60^\circ = S120^\circ/S60^\circ = K''\left[1 - \frac{3}{2}\left\{3C_0 - C_\varphi\left(1 - \frac{\lambda f'_\varphi}{3f_\varphi}\right)\right\}\frac{\delta\lambda 90^\circ}{\lambda}\right] \ldots \quad (12)$$

For a light scatterer like paraffin for which interference in the direction $\phi = 60^\circ$ is small and so also its rate of variation with λ, (vide Pal 1948) the term $\lambda f'_\varphi/3f_\varphi \ll 1$, so that the above ratio reduces to

$$K''\left[1 - \frac{3}{2}\{3C_0 - C_\varphi\}\frac{\delta\lambda 90^\circ}{\lambda}\right]$$

C. *The discontinuity* :

Equations (1) and (12) respectively show that the intercepted ratios $(S'/P')90^\circ$ and $(S'120^\circ/S'60^\circ)$ are each a function of C i.e. of the percentage modification of the scattered rays in the direction or directions concerned. Consequently any perturbation of this quantity is expected to affect the ratio. If C increases the ratio suffers a diminution.

A small change in C, of the order of a few per cent only is, however, not likely to upset appreciably the balance in equations (7) and (8), which happen to be the conditions for horizontality of the graph in (12); for, a variation in C is attended with a variation in Δ_φ or p_x in the same sense.

A sudden small increase occurring in the C-value during experiment can only mean an abrupt discontinuity appearing in the graph concerned, at the proper place there being no change in the nature of the same, except that its subsequent course is bodily lowered down. Obviously, such an occurrence is detectable by its effect on absorption.

Yet the origin of this kind of discontinuity, according to the present view, is fundamentally in the material of the scatterer and not in the absorber, as may be supposed on the evidence furnished by the mass-absorption coefficients.

The discontinuities observed by Barkla with Khastgir, Mackenzie and others may be explained in the manner suggested above.

EVIDENCES AND DISCUSSION
GENERAL CONSIDERATIONS

From what has already been said it will appear that the discontinuity arising from a sudden variation of C i.e., of the percent modification on scattering is essentially associated with the scattered radiation. One must therefore look to the scattered radiation alone for the realisation of this kind of discontinuity.

Let us now examine the soundness of the idea put forward. The percentage modification in Compton scattering is determined by the number of free electrons which recoil from the parent atom due to the impact of the incident photons. When high-energy quanta, generated at high exciting voltage are scattered, practically all the loosely bound electrons are ejected from the atom, with the result that modification is complete i.e., cent per cent. On the contrary, at low exciting voltages the energy of the colliding photon is wholly inadequate to release any electron from the atom, so that the scattered rays are unmodified. With medium exciting voltage, however, only a few of the electrons are emitted. This means a partial modification of the scattered rays. It is in this case, that under the action of continued impulses from the incident photons, a few more electrons than normally* at the outset, may be expected to recoil from the scattering atom in a given time, particularly when it is a light one. In addition, the possibility may not be ruled out that some slight and appropriate changes of a casual nature, occurring in the microscopic structure of the primary radiation itself may also contribute to the same effect.** In the circumstances, one may expect a slightly greater percentage modification than previously. It thus appears reasonable that such a process may occasionally be in actual operation, suddenly stepping up the modification by a few per cent .A reversion due to 'fatigue' may also be conceived.

This concept gets strong support from the experimental observation of Barkla and Kay (1933) that medium exciting voltages around 60KV(peak) were the most favourable for the occurrence of the J-discontinuity—a fact which was confirmed by the writer also (unpublished). It should be mentioned here, that the discontinuities in the experiments of Barkla and Watson (1926) were also obtained with 60KV. (peak), while those in Barkla and Mackenzie's (1926a) with about 40KV. (peak).

---

*It may as well be that, owing to some kind of 'inertia' or lag, the normal number of electrons may not, at times be emitted till the atom has been irradiated for some time depending on the energy of the incident quanta.

**Barkla and Mackenzie (1926b) found that a certain frequency of interruption of the induction coil used for generating high voltage applied to the X-ray tube was more favourable for the occurrence of the J-discontinuity than any other frequency of interruption, greater or less. The composition of the primary beam from the X-ray tube is determined to some extent by the rate of interruption of the induction. The rate of interruption may therefore create a condition favourable or otherwise for the discontinuity to occur.

The intermediate frequency of interruption of the induction coil which was found most favourable for the J-discontinuity of the second kind to appear, is also consistent with the observation that such discontinuities are usually obtained with medium exciting voltages.

(a) *Multiple discontinuities*:

Granting the possibility of a sudden change (increase) in the C-value under favourable circumstances, there is no reason why more than one such change should not be admissible in an unsteady state. Each change will correspond to a discontinuity. In this way, several discontinuities, named J_1, J_2, ... have been actually observed in course of a single experiment. For example, Barkla and Watson reported as many as four discontinuities, whereas Barkla and Mackenzie obtained generally upto two. With more extensive filtration of the beams concerned, the number might increase to three.

(b) *Drop in the ratio*:

In the filtering experiments of Barkla and Mackenzie performed with aluminium and paraffin scatterers, (*vide* Barkla and Mackenzie, 1926a, Fig. 4, p. 547), the drop in the ratio at each discontinuity, was about 7.5% for aluminium and about 10% for paraffin. The greater drop for paraffin is very significant, being another evidence in support of the idea advanced. For, paraffin being lighter than aluminium, the probability of modification produced by the former should be greater than that produced by the latter; so also the probability of any fluctuation of the same*. That the percentage modification diminishes with an increase in the (i) atomic number of the scattering element and (ii) incident wavelength, was experimentally verified by Backhurst (1934) at $\phi = 150°$ and also by others.

According to Barkla, the scattered beam in the direction, $\theta = 120°$, was mainly responsible for the abrupt fall in the ratio at the discontinuity in the experiment of Barkla and Mackenzie (1926a). This would correspond to a variation in C_θ causing the major portion of the fall. It should, indeed, be so, according to our analysis also. (*Vide* eqn. 12).

(c) *The shift of the discontinuity in progressive filtering experiments* :

Barkla and Mackenzie (1926a) and later Barkla and Kay (1933) found that the effect of softening the primary beam was to shift the discontinuity towards a higher value of x; that is to say, for the discontinuity to be observed, filtering had to proceed a little further than previously. The reverse (i.e., a shift of the discontinuity towards a lower value of x)—was found when the primary beam was hardened. The shift to a higher value of x may be regarded as due to a longer exposure necessary for the softer radiation (at the lower voltage) to excite the scattering atom into that critical state at which a few extra or more electrons are ejected per unit time. This is because, a softer radiation means lesser average

*For the same reason again, the probability of *occurrence* of a C-change should be greater for paraffin-scatterer than for aluminium. This was substantiated by the experiments of Barkla and Mackenzie and also of Barkla and Watson, where paraffin usually gave a greater number of discontinuities than aluminium. (*Vide* Barkla and Mackenzie, 1926 a, Fig. 3., p. 545).

energy for the incident quanta. The required longer exposure generally carries the experimenter through a greater distance from the origin in the graph. With a more penetrating radiation— say a filtered one —a shorter exposure is expected to bring about the necessary condition, resulting in an earlier occurrence of the discontinuity, if it occurs at all. The same remark holds good generally irrespective of the sequence in which the different exciting voltages are applied, provided they are of the right magnitude. (*Vide* Barkla and Kay, 1933, Fig. 6, p. 471).

(d) *Critical absorbability*

With regard to the critical absorbabilities, at which the J-discontinuities of the second kind were expected to appear, it may be said that the values obtained by different investigators and under different experimental conditions* reveal a wide divergence amongst themselves, thereby detracting much from the 'critical' character of the quantity, which should have been, otherwise, well-defined. For instance, obtained with aluminium absorber, $(\bar{\mu}/\rho)$ for J_2 varied from 1.8. (Barkla and Khastgir) to 2.2 (Barkla and Kay). Watson got figures higher still (2.35). This works out to a divergence from the mean value, of about 10% or even more. $(\bar{\mu}/\rho)_{Al}$ for J_1 had been observed to range from *mean* 3.6 (Barkla and Mackenzie) to 3.8 (Barkla and Khastgir), which means a divergence of about 6%. The divergence for J_3 is also of the same order. These divergences certainly far exceed the limits of accuracy—usually about $\pm 2\%$ (Barkla and Mackenzie, 1926a) claimed for the absorption coefficient in this type of experiments. Furthermore, as many as eight discontinuities have to be accommodated within a span extending from $(\bar{\mu}/\rho)_{Al}$ = .34 to $(\mu/\rho)_{Al}$ =3.8, the corresponding wavelengths being 0.23 Å.U. and 0.63 Å.U respectively. The so-called 'critical' mass-absorption coefficients are, therefore, deprived of a considerable portion of the weights attached to them. According to the present view, on the contrary, the above discontinuities are the outcome of random fluctuations in the electron-emission from the scattering atom—fluctuations which apparently know no law except the law of chance. The proximity to certain values of $(\bar{\mu}/\rho)_{Al}$ (in the region of medium wavelength) however, is more conducive to the initiation of the process suggested.

The discontinuity J_3 in the light of this analysis, also belongs to a different category and was discussed elsewhere, (Pal, 1964).

The chance character of the discontinuity is more convincingly brought out by the appearance of new discontinuities between the so-called J_1 and J_2 and between J_2 and J_3 in the experiments of Barkla and Watson (1926).

(e) *Occurrence and non-occurrence of a discontinuity*:

A very interesting and yet puzzling aspect of the J-phenomenon had been brought to light by experiments of Barkla and Kay (1933). They took two scatterers A and B, made of the same material (filter paper or paraffin wax) and alike

*Large variations with variations in the tube current were recorded by Watson (1926).

in dimensions. A was an old scatterer which was in use for a long time past and B, a newly-made one, previously unexposed to radiations. The two specimens A and B were then irradiated and observations made, as usual, sometimes. alternately on the same day. Specimen B of filter paper and A of paraffin wax consistently exhibited the discontinuity day after day, while no discontinuity was observed with the other specimen of the same material. This state of affairs persisted for quite a long time, till the scatterer hitherto giving the discontinuity all right, also ceased to give it any longer. With a view to make the test as stringent as possible. the experiment was repeated by placing the two specimens A and B of paraffin wax in the same position alternately, corresponding to each absorbing thickness of the series. But this made no difference in the nature of the result previously observed.

Obviously, a behaviour of this kind cannot be reconciled with any probable variation in the radiation alone. The unescapable presumption therefore, is that so far as the occurrence of the discontinuity is concerned, it is intimately linked up with the scattering process and yet the chemical composition of the scatterer is not the only deciding factor. Something more is needed. It may be some particular 'state' of or in the scatterer as suggested by Barkla; the state itself being dependent on or dominated by the whole associated history.

The evidence cited, does not upset the hypothesis of extra electron-emission, if it is conceded that two scatterers though chemically and in all external appearances identical, may yet differ substantially in their reaction towards X-ray stimuli; though in what way exactly, it is not possible to say. The necessary responsiveness was actually present, in the beginning, in one specimen but never in the other. This responsive feature again was not of a permanent character, in as much as it disappeared after some time. Neither could the irresponsive scatterer, though in use all the while, be thrown into the hypersensitive state in which a few more electrons could recoil per unit time, even under continued exposure. It thus appears that neither previous over-exposure nor absolute non-exposure is decisive. The precise nature of the influence that helps or prevents the necessary peculiar disposition of the scatterer is yet unknown.

(f) *The C-levels*:

When the percentage of modification i.e. C has undergone a change, it may or may not settle down to its new value. There may be further sudden and localized changes including even a reversion to the old value. The experiments of Barkla and Watson and also of Barkla and Mackenzie bear testimony to the former kind of behaviour and our experience with a Müller tube (hot-cathode, water-cooled) to the latter. Instances are also on record, depicting how the plots on the graph, systematically choose between two (or amongst three) distinct levels indicating preferred values for C. In this connection, we can refer to our Fig. 1

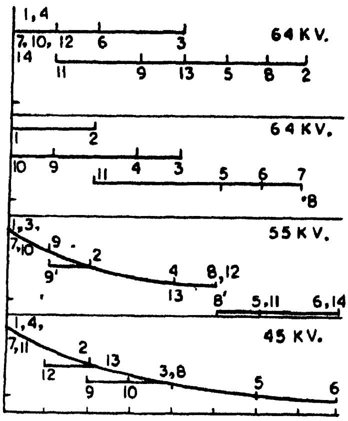

Fig. 1. Illustrating the fluctuation of percentage modification of the scattered X-rays in the unsteady state.

and to the results of Barkla and Khastgir (1925) (*Vide* Figs. 2, 3 and 4, pp. 1119-1123) where the stratified graphs illustrate how C changed by jumps between well-defined levels at the same wavelength. In this context, some truth may be sensed in the postulate of 'levels' of X-ray activity with which Barkla associated the J-phenomenon. The J-levels may be identified with the C-levels, in view of the fact that an increase in C is attended with a greater absorption of the scattered rays.

(g) *Superposition of X-rays*:

Barkla and Mackenzie (1926a) and later Barkla and Sen Gupta (1929) performed experiments with one beam of X-rays superposed on another by which they sought to establish that the J-absorption and all the associated phenomena depended on the whole of a complex beam and not on individual harmonic constituents of the radiation considered separately. This view, however, was not established. We shall outline here the experiments of Mackenzie and of Sen Gupta with Barkla and the results obtained by them. We shall also give an explanation which seems satisfactory and consistent.

(i) Experiments of Barkla and Mackenzie on progressive filtering of the two beams.

The position of the J-discontinuity was at first found with a thin sheet of aluminium acting as a scatterer. On interposing a second thicker sheet of the same material behind and in contact with the first, so that the rays scattered by it, were superposed on those scattered by the first scatterer, it was found that the discontinuity was displaced to the left, i.e. toward a lower value of the thickness x of the filter.

The result of these experiments can be explained as follows : After the second sheet had been interposed, the incident radiation being filtered through deeper layers, of the scatterer became, on the average, more penetrating than previously and hence the incident quanta became more energetic. The discontinuity if it appears, should therefore, appear earlier.

(ii) Experiments of Barkla and Sen Gupta.

Two scattering slabs of paraffin wax —R_1 and R_2 —were placed in parallel positions separated by a distance in the path of the primary rays. The scattered radiation from R_2 could be superposed whenever desired, on that from R_1, upon and inside the absorbing sheets A_2 only. In the experimental set-up, it could be so arranged that the primary rays were received by (i) R_2 first and (ii) R_1 first. (*Vide* Barkla and Sen Gupta, 1929, Figs. 1 and 2).

On superposition it was observed that the discontinuity was displaced to the right in case (i) and to the left in case (ii).

The superposed scattered radiation from R_2, in each case, evidently could not directly affect the ionization inside the ionization chamber due to the radiation scattered from R_1 except through a tertiary contribution due to scattering by A_2. This tertiary contribution must necessarily have been of a negligible magnitude. This fact was borne out by the plots on the graph being actually identical with the superposed beam 'on' or 'off', except near the discontinuity. In the circumstances, it is not quite clear why there should be an exception with regard to the discontinuity itself, if it had fundamentally originated in an absorption process.

We are, however, inclined to the view that, as in Barkla and Mackenzie's experiments, here also, the observed fact was not a superposition effect. It is suggested that in the experiments of Barkla and Sen Gupta, the position of the discontinuity and also the number of such discontinuities were determined solely by chance. The appearance of a second discontinuity to the right of the original one, besides the one to the left (*Vide* Barkla and SenGupta, 1929, Fig. 3, graph 2) suggests a random character of the phenomenon.

(h) *The unsteady state*

A casual variation in C, as described above, should always be recognized as an unsteady state of affairs departing from the ideal. A steady C yields a continuous curve, whereas abrupt deviations in its value give rise to the well-known steps or discontinuities, whatever the degree of heterogeneity of the rays concerned.

In further support of what has been stated in connection with abrupt fluctuation of C, a few illustrative samples of graphs, obtained by the present writer (with Al* as the absorbing substance) are furnished in Figure 1. They speak for themselves. The numbers assigned to the plots show the sequence in which observations were made. It should be pointed out here, that any change in C

*The purity of Al. used was not chemically tested.

should not theoretically affect the unintercepted ratio. i.e. $(S'/P')90^\circ$ corresponding to $x = 0$, as will be evident from Eqn. (1). In practice, however, even when $x = 0$ the rays have yet to pass through one foil of Al ($x = 0.1$ mm) which covers the window of the ionization chambers. A zero thickness in the graph is really a thickness of 0.1 mm. Other thicknesses are to be amended accordingly.

SUMMARY AND CONCLUSION

The ratio of ionizations, $S'120^\circ/S'60^\circ$, as a function of the thickness x of the absorbing substance placed in each beam, has been theoretically obtained and the condition of horizontality of the graph showing $S'120^\circ/S'60^\circ$ against x has been deduced. The experimental results showing the graph to have a downward slope in the direction of increasing thickness or to be a horizontal straight line parallel to the x-axis, have thus been explained. The discontinuity observed by Barkla and Mackenzie (1926a) in this graph or in the $(S'/P')90^\circ - x$ graph of Barkla and Khastgir (1925a) has been ascribed to an abrupt increase in the percentage modification associated with the Compton scattering*. The sudden variation of the proportion of the modified rays has been attributed to a casual excess electron-recoil from the scattering atom. Such a discontinuity is, therefore, of a second kind, as distinguished from that of the first kind discussed in a previous paper (Part I, Pal, 1964). It may be recalled that the discontinuity of the first kind was shown to be linked with what was called the 'D-point' which was governed by the character of the heterogeneous radiation concerned and the nature of the absorbing substance.**

The concept of excess electron-recoil successfully explains (i) the necessity of medium voltage of excitation of X-rays for the occurrence of the discontinuity of the second kind, (ii) the occurrence of multiple discontinuities, (iii) the greater drop in the ratio, $S'120^\circ/S'60^\circ$ or $(S'/P')90^\circ$, and a greater number of such discontinuities in a lighter substance (paraffin) than for a relatively heavier substance (aluminium) used as scatterer and (iv) the shift of the discontinuity towards a higher or a lower value of x, according as the beam is less or more penetrating.

As the values obtained experimentally for the critical absorbability at which a J-discontinuity of the second kind was expected to occur, showed a wide range, the critical character of the mass-absorption coefficient losses its significance and it is believed that the position of such a discontinuity is only a matter of chance.

---

*If there is truth in this concept, it seems probable that in suitable circumstances, the associated discontinuity should also appear with monochromatic X-rays.

**The discontinuity of the first kind may be obtained with scattered radiations also. An abrupt change in the C-value, depending on the casual excess electron-recoil during the scattering process and causing a discontinuity of the second kind may have an indirect influence on the 'D-point'. This complication may perhaps account for the capricious appearance and disappearance of the J-discontinuity (first kind) in the case of scattered radiations under apparently identical experimental Conditions—(*Vide Pal,* 1964, p. 76).

The puzzling experimental results of Barkla and Kay (1933) regarding the occurrence and non-occurrence of a discontinuity of the second kind, obtained with an old and a newly-made scatterer seem to suggest that the chemical composition of the scatterer is not the only deciding factor : *a condition* of matter determined by its previous history and also the composition and character of the radiation depending to some extent on the frequency of interruption of the induction coil appear to play an important role.

The J-absorption 'level' postulated in this connection by Barkla receive in a restricted sense a possible interpretation in the concept of the fluctuation of the electron-emission from the scattering atom. Also the element of uncertainty involved in the process explains the *dubious* character of the J-discontinuity of the second kind.

The effect of superposition of two beams of X-rays as observed by Barkla and Mackenzie (1926a) and also by Barkla and Sen Gupta (1929) has also been discussed and interpreted.

ACKNOWLEDGMENT

The author wishes to acknowledge his indebtedness to late Prof. C. G. Barkla, F.R.S., N.L., for affording him all facilities for experimental research in his laboratory at Edinburgh University and for giving him helpful guidance during the period 1935-37. His best thanks are also due to Prof. S. R. Khastgir, D.Sc., F.N.I., Head of the Dept. of Physics, Bose Institute, Calcutta, for valuable discussions.

REFERENCES

Backhurst, I., 1934, *Phil. Mag.* **XVII**, 321.
Barkla, C. G., 1918, *Phil. Trans. Roy. Soc., A*, **CCXVII**, 315.
——— 1924, *Nature*, **CXIV**, 753.
——— 1925, *Phil. Mag.*, **XLIX**, 1033.
——— 1926, *Nature*, **CXVII**, 448.
——— 1927, *Nature*, **CXIX**, 778.
——— 1928, *Phil. Mag.* **V**. 1164.
——— 1929, *International Critical Tables*, **VI**, 1.
——— 1931, *Nature*, **CXXVII**, 877.
——— 1933, *Nature*, **CXXXI**. 166.
Barkla, C. G., and Kay, J. S., 1933, *Phil. Mag.* (Suppl. Aug.) **XVII**, 457.
Barkla, C. G., and Khastgir, S. R., 1925a, *Phil. Mag.* **XLIX**, 251.
——— 1925b, *Phil. Mag.* **I**, 1115.
——— 1926, *Nature*. **CXVII**, 228.
——— 1926, *Phil. Mag*, **II**, 642.
——— 1927, *Phil. Mag.* **IV**, 735.
Barkla, C. G., and Mackenzie, G. I., 1925, *Nature*, **CXV**, 942.
——— 1926a, *Phil. Mag.* **I**, 542.
——— 1926b, *Phil. Mag.* **II**, 1116.

Barkla, C. G., and Sen Gupta, M. M., 1929, *Phil. Mag.* **VII**, 737.
Barkla, C. G., and Watson, W. H., 1926, *Phil. Mag.* **II**, 1122.
Khastgir, S. R., and Watson, W. H., 1925, *Nature*, **CXV**, 604.
——— 1925, *Nature*, **CXVI**, 47.
Khastgir, S. R., 1932, *Phil. Mag.* **XIV**, 99.
Pal, H. K., 1948, *Ind. Jour. Phys.* **22**, 291.
——— 1949, *Ind. Jour. Phys.* **23**, 111.
——— 1964, *Ind. Jour. Phys.* **38**, 61.
——— 1965, *Ind. Jour. Phys.* **39**, 108.
Watson, W. H., 1925, *Proc. Roy. Soc.* (Edinburgh) **XIV**, 48.
——— 1928, *Phil. Mag.* **V.**, 1146.